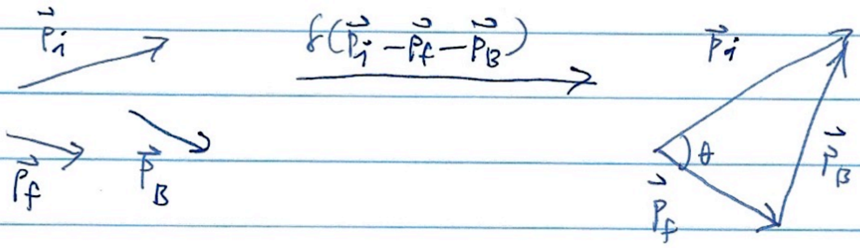
Schwartz 5.1

$$d\Pi = (2\pi)^4 \delta(\Sigma p) \frac{d^3 p_f}{(2\pi)^3 2E_f} \frac{d^3 p_B}{(2\pi)^3 2E_B}$$

$$= \frac{1}{E_f E_B} \frac{1}{\cancel{4}16\pi^2} \delta(E_f + E_B - E_i) \delta(\vec{p}_i - \vec{p}_f - \vec{p}_B) d^3p_f \, d^3p_B$$



The d^3p_B is integrated over to close the triangle.

$$x = E_f + E_B - E_i = \sqrt{|\vec{p}_f|^2 + m_f^2} + \sqrt{|\vec{p}_B|^2 + m_B^2} - E_i$$

Apply law of cosine, it is

$$\sqrt{|\vec{p}_f|^2 + m_f^2} + \sqrt{|\vec{p}_i|^2 + |\vec{p}_f|^2 - 2|\vec{p}_i||\vec{p}_f|\cos\theta + m_B} - E_i .$$

With triangle closed, we have

$$d\Pi = \frac{1}{E_f E_B} \frac{1}{\cancel{4}16\pi^2} \delta(x) |\vec{p}_f|^2 \, d\theta \, d|\vec{p}_f|$$

$$\frac{dx}{d|\vec{p}_f|} = \frac{|\vec{p}_f|}{E_f} + \frac{|\vec{p}_f| - |\vec{p}_i|\cos\theta}{E_B} \Rightarrow d|\vec{p}_f| = dx \left[\frac{|\vec{p}_f|}{E_f} + \frac{|\vec{p}_f| - |\vec{p}_i|\cos\theta}{E_B} \right]^{-1}$$

$$\left[\frac{|\vec{p}_f|}{E_f} + \frac{|\vec{p}_f| - |\vec{p}_i|\cos\theta}{E_B}\right]^{-1} = E_f E_B \left[E_B |\vec{p}_f| + E_f (|\vec{p}_f| - |\vec{p}_i|\cos\theta)\right]^{-1}$$

$$\Rightarrow d\Pi = \frac{1}{\cancel{E_f}\cancel{E_B}} \frac{1}{16\pi^2} \delta(x) |\vec{p}_f|^2 d\theta \frac{\cancel{E_f}\cancel{E_B}}{1} \left[E_B|\vec{p}_f| + E_f(|\vec{p}_f| - |\vec{p}_i|\cos\theta)\right]^{-1} dx$$

$$= \frac{1}{16\pi^2} |\vec{p}_f|^2 \left[E_B |\vec{p}_f| + E_f(|\vec{p}_f| - |\vec{p}_i|\cos\theta)\right]^{-1} d\theta$$

$$d\sigma = \frac{1}{2E_1 2E_2 |\vec{v}_1|} |\mathcal{M}|^2 d\Pi$$

$$= \frac{1}{64\pi^2 m_A} \frac{|\vec{p}_f|}{|\vec{p}_i|} \left[E_B + E_f \left(1 - \frac{|\vec{p}_i|}{|\vec{p}_f|}\cos\theta\right)\right]^{-1} d\theta$$

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2.27.2024